UNIT-4 : NETWORK PROBLEMS AND PROJECT MANAGEMENT

Network Scheduling (Project management) is a technique used for planning and scheduling large projects, in the field of construction, maintenance, fabrication and purchasing of computer systems, etc.

Basic planning and controlling techniques that utilize a network to complete a predetermined project or schedule.

1. Programme Evaluation Review Technique (PERT)
2. Critical Path method (CCPM)

Project :- A project is defined as a combination of interrelated activities, all of which must be executed in a certain order for its completion.

Three phases of project :

- Planning
  * Setting the objectives of the project
  * Assumptions to be made
  * Listing of tasks or jobs that must be performed in order to complete a project under consideration.
  * The estimates of costs and duration of the various activities, the manpower, machines & materials required for the project are also determined.

- Scheduling.

  This consists of laying the activities according to their order of precedence and determining the following

  (i) The start & finish times for each activity.

  (ii) The critical path on which the activities require special attention.

  (iii) The slack & float for the non-critical paths.

- **Controlling** :- This involves the following :
  (i) Making periodical progress reports
  (ii) Reviewing the progress
  (iii) Analyzing the status of the project
  (iv) Making management decisions regarding updating, crashing & resource allocation, etc.

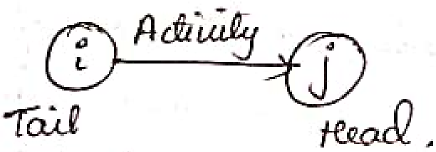
Basic terms

Network :- It is the graphical representation of logically & sequentially connected arrows & nodes, representing activities and events in a project.
(It also called 'arrow diagram')

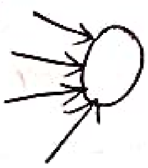
Activity :- An activity represents some action & is a time consuming effort necessary to complete a particular part of the overall project.

Here A is called the activity.

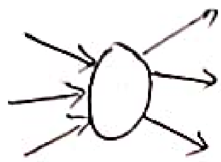
Event :- The beginning & end points of an activity are called Events or nodes.



Merge event :- When more than one activity comes and joins an event, such event is known as merge event.



Burst event :- when more than one activity leaves an event, such event is known as burst event.



Merge & burst event.

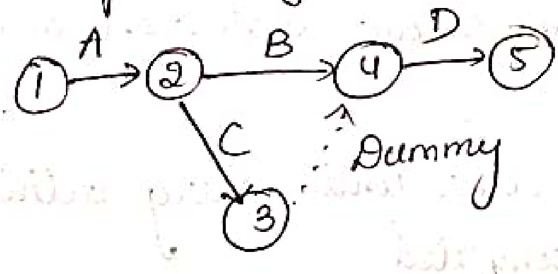
Activities can be further divided into four categories.

(i) Predecessor activity - An activity which must be completed before the start of one or more other activities is known as predecessor activity.

(ii) Successor activity - An activity which starts immediately after the completion of one or more of other activities is known as successor activity.

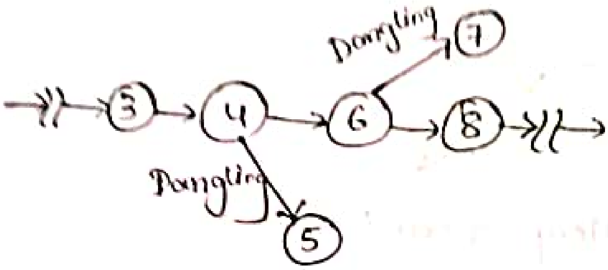
(iii) Concurrent activity - An activity which can be accomplished concurrently is known as concurrent activity.

(iv) Dummy activity - An activity which does not consume either any resource and/or time is known as dummy activity.

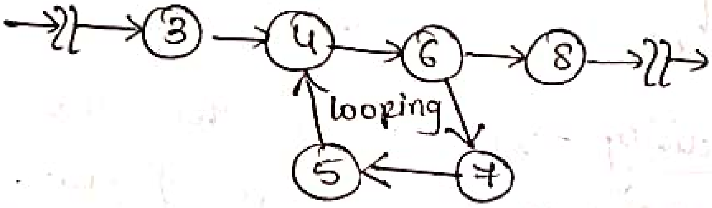


Common Errors in a network construction.

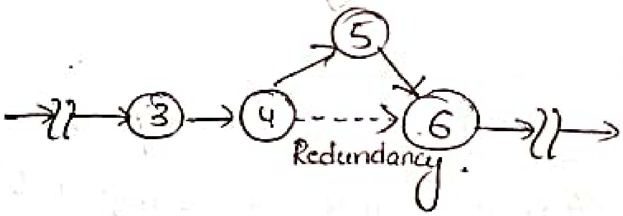
1. **Dangling** - If an activity is disconnected before completion of all activities in a network diagram then such an error is called dangling error.



2. **Looping (or Cycling)** - Endless loop in a network diagram is known as error of looping.



3. **Redundancy** - If a dummy activity is inserted in a network unnecessarily then that error is called redundancy error.



RULES OF NETWORK CONSTRUCTION.

(i) Try to avoid the arrows that cross each other.

(ii) Use straight arrows.

(iii) No event can occur until every activity preced it has been completed.

(iv) An event cannot occur twice, i.e., there must be no loops.

(v) An activity succeeding an event cannot be started until that event has occured.

(vi) Use arrows from left to right. Avoid mixing two directions, vertical & standing arrows may be use, if necessary.

(vii) Dummies should be introduced only, if it is extremely necessary.

(viii) The network has only one entry point called the start event & one point of emergence called the end or terminal event.

Imp. NUMBERING THE EVENTS (FULKERSON'S RULE)

After drawing the network in a logical sequence, every event is assigned a unique number.

The number sequence must be such so as to reflect the flow of the network.

(i) Event numbers should be unique.

(ii) Event numbering should be carried out on a sequential basis, from left to right.

(iii) The initial event, which has all outgoing arrows with no incoming arrow is numbered as 1.

(iv) Delete all the arrows emerging from all the numbered events. This will create atleast one new start event, out of the preceding events.

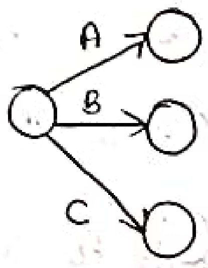
(v) Number all new start events 2, 3 & so on. Repeat this process until the terminal event without any successor activity is reached. Number the terminal node suitably.

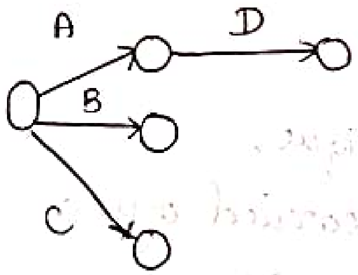
Problems:-

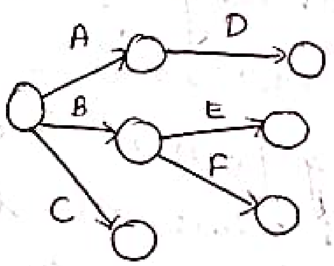
Q1. Construct the network for each of the projects whose activities and their precedence relationships are given as :

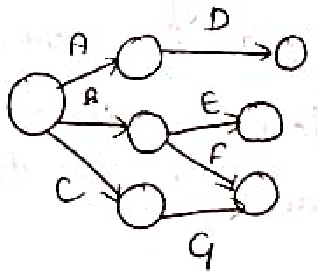
Activity	A	B	C	D	E	F	G	H	I	J	K
Immediate predecessor	–	–	–	A	B	B	C	D	E	H, I	F,

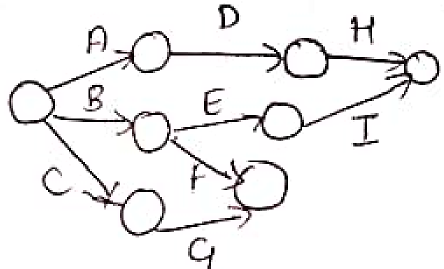
J&K will be the ending activities bcz it is not predecessor of any.

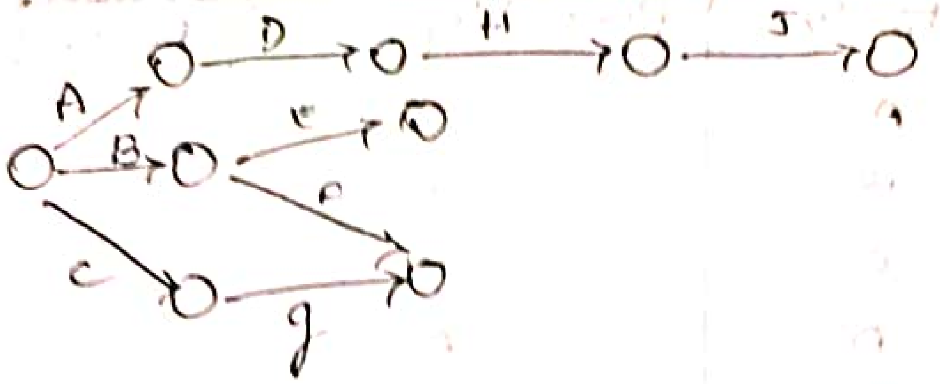




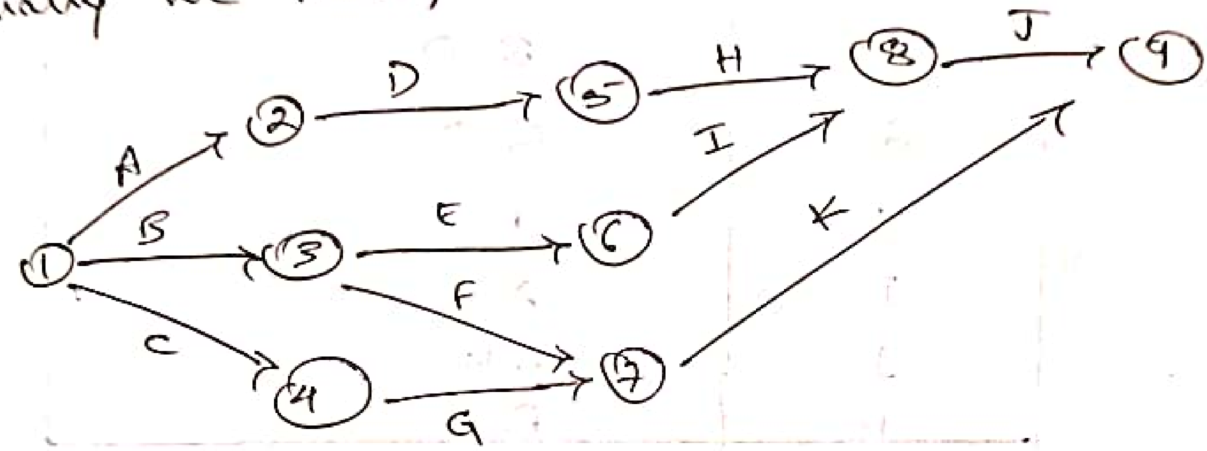






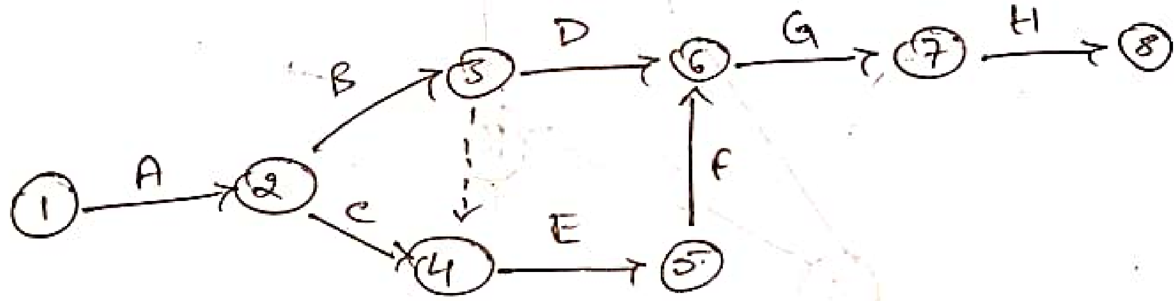


Finally we have,



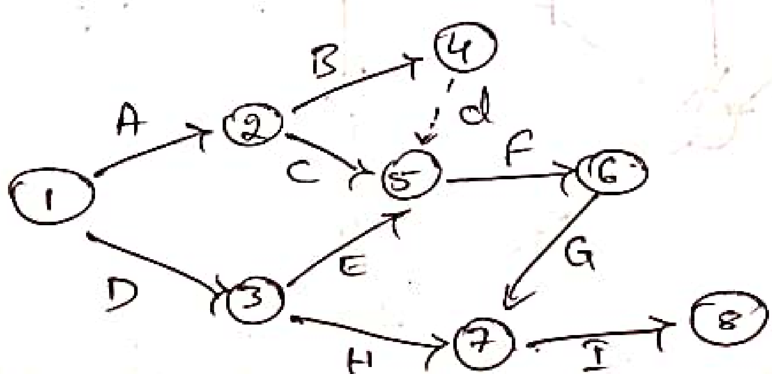
22.

Activity	A	B	C	D	E	F	G	H
Predecessor	-	A	A	B	B,C	E	D,F	G



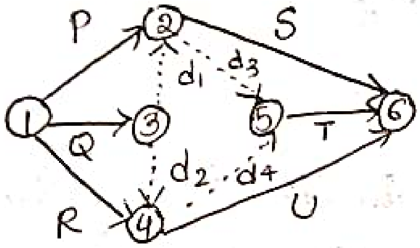
23.

Activities	A	B	C	D	E	F	G	H	I
Preceding Activities	-	A	A	-	D	B,C, E	F	D	G, H



Q5. Draw the network for the project whose activities and their precedence relationships are given below.

Activity	P	Q	R	S	T	U
Predecessor	-	-	-	P, Q	P,R	Q, R



d_1, d_2, d_3, d_4 are dummy activities.

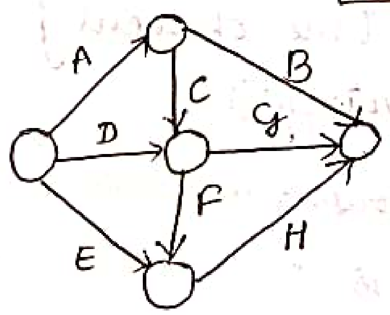
Q6. Construct the network for the project whose activities and their relationships are given below:

Activities: A, D, E can start simultaneously.

B, C > A ; G, F > D, C ; H > E, F.

Sol:-

A	B	C	D	E	F	G	H
-	A	A	-	-	D, C	D, C	E, F



Start activities are A, D, E.

End activities are H, G, B

Critical Path Method (CPM)

The Objective of critical path analysis is to estimate the total project duration and to assign starting and finishing times of all activities involved in the project.

This helps in checking actual progress against the scheduled duration of the project.

* An activity in a network diagram is said to be critical, if the delay in its start will further delay the project completion time.
* A non critical activity allows some scheduling slack, so that the start time of the activity may be advanced or delayed within limits without affecting the completion date of the entire project.

Terms used:-

E_i - Earliest Occurance time of event i

L_j - Latest Occurance time of event j

t_{ij} - Duration of activity (i,j)

We shall use the following notation for basic scheduling computations.

(i,j) = Activity (i,j) with tail event i & head event j

T_{ij} = Estimated completion time of activity (i,j)

ES_{ij} = Earliest starting time of activity (i,j)

EF_{ij} = Earliest finishing time of activity (i,j)

LS_{ij} = Latest starting time of activity (i,j)

LF_{ij} = Latest finishing time of activity (i,j)

Forward Pass computations (for Earliest Event time)

Step 1:- The computations begin from the start node and move towards the end node. Let zero be the starting time for the project.

Step 2:- Earliest starting time $(ES_{ij}) = E_i$ is the earliest possible time when an activity can begin, assuming that all of the predecessors are also started at their earliest starting time. Earliest finish time of activity (i,j) is the earliest starting time + activity time.

$$(EF)_{ij} = (ES)_{ij} + t_{ij}$$

Step 3:- Earliest event time for event j is the maximum of the earliest finish time of all the activities, ending at that event.

$$E_j = \max_i (E_i + t_{ij})$$

The computed 'E' values are put over the respective rectangles representing each event.

Backward Pass computations (for latest Allowable Time)

Step 1:- For ending event assume, $E = L$.

Step 2:- Latest finish time for activity (i,j) is the target time for completing the project.

$$(LF)_{ij} = L_j$$

Step 3:- Latest starting time of the activity (i,j) = latest completion time of (i,j) − the activity time

$$LS_{ij} = LF_{ij} - t_{ij}$$
$$= L_j - t_{ij}$$

Step 4:- Latest event time for event i is the minimum of the latest start time of all activities originating from the event.

$$L_i = \min_j (L_j - t_{ij})$$

The computed 'L' values are put over the respective triangles representing each event.

Determination of Floats and slack times.

- **Float** is defined as the difference between the latest and the earliest activity time.
- **Slack** is defined as the difference between the latest and the earliest event time.

KINDS OF FLOATS:

- **Total float:-** The total float of an activity (i,j) is the difference between the latest start time & the earliest start time of that activity.

$$(TF)_{ij} = (LS)_{ij} - (ES)_{ij}$$

$$\text{or } (TF)_{ij} = (L_j - E_i) - t_{ij}$$

- **Free float:-** The free float of an activity (i,j) denoted by $(FF)_{ij}$ is the time by which the completion of an activity can be delayed beyond the earliest finish time, without affecting the earliest start of a subsequent succeeding activity.

$$(FF)_{ij} = (E_j - E_i) - t_{ij}$$

$(FF)_{ij}$ = Total float - Head event slack

Head event slack $= L_j - E_j$

- **Independent float :-**
The amount of time by which the start of an activity can be delayed, without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time.

$$IF_{ij} = (E_j - L_i) - t_{ij}$$

or

$$(IF)_{ij} = \text{Free float} - \text{Tail event slack}.$$

Tail event slack $= L_i - E_i$

* The negative independent float is always taken as zero.

$$\boxed{IF_{ij} \leq FF_{ij} \leq TF_{ij}}$$

Critical activity :-
An activity is said to be critical, if a delay in its start cause a further delay in the completion of the entire project.

Critical path :- The sequence of critical activities in a project network which determines the duration of a project is called the critical path.

In the network it is denoted by a double line & identifies all the critical activities of the project. Hence, for the activities (i,j) to lie on the critical path, following conditions must be satisfied.

(a) $ES_i = LF_i$

(b) $ES_j = LF_j$

(c) $ES_j - ES_i = LF_j - LF_i = t_{ij}$

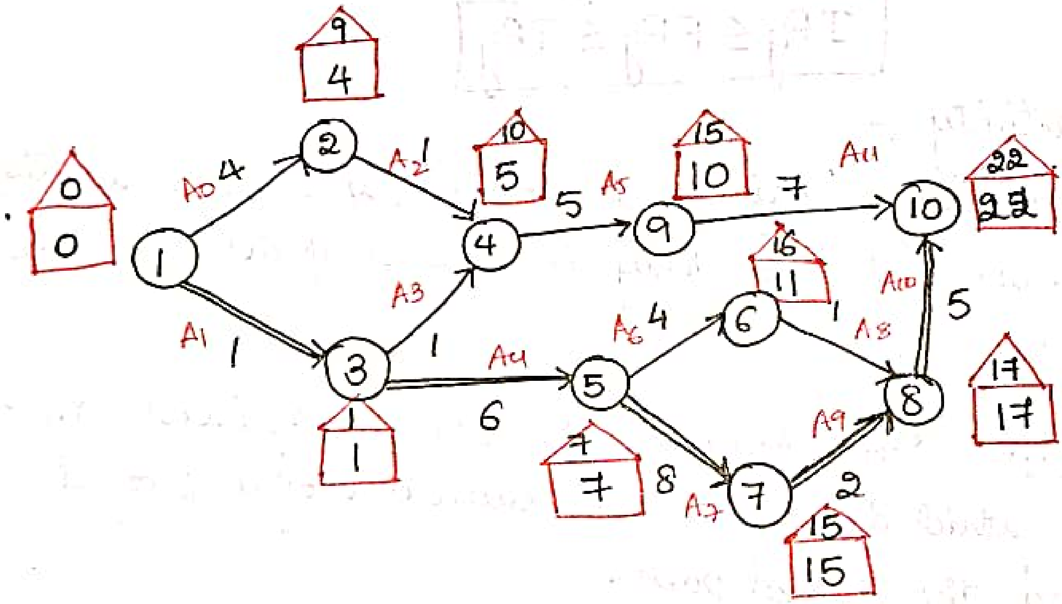
- ES_i & ES_j are the earliest start & finish time of events i & j.
- LF_i & LF_j are the latest start & finish time of the events i & j.

Q. A project schedule has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	[illegible]
Time (days)	4	1	1	1	6	5	4	8	1	2	5	[illegible]

From the above information, you are required to:
1. Construct a network diagram.
2. Compute the earliest event time and latest event time
3. Determine the critical path and total project duration
4. Compute total & free float for each activity.

Sol:- First we construct the network



Forward Pass method.

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0+4 = 4$

$E_3 = E_1 + t_{1,3} = 0+1 = 1$

$E_4 = \max\{E_2 + t_{2,4} ; E_3 + t_{3,4}\}$
$= \max\{4+1 ; 1+1\}$
$= \max\{5, 2\}$
$= 5$

$E_5 = E_3 + t_{3,5} = 1+6 = 7$

$E_6 = E_5 + t_{5,6} = 7+4 = 11$

$E_7 = E_5 + t_{5,7} = 7+8 = 15$

$E_8 = \max\{E_6 + t_{6,8} ; E_7 + t_{7,8}\}$
$= \max\{11+1 ; 15+2\}$
$= \max\{12 ; 17\}$
$E_8 = 17$

$E_9 = E_4 + t_{4,9} = 5+5 = 10$

$E_{10} = Max\{E_8 + t_{8,10} ; E_9 + t_{9,10}\}$

$= Max\{17+5 ; 10+7\}$

$= Max\{22 ; 17\}$

$= 22$

Backward pass method.

$L_{10} = E_{10} = 22$

$L_9 = L_{10} - t_{9,10} = 22 - 7 = 15$

$L_8 = L_{10} - t_{8,10} = 22 - 5 = 17$

$L_7 = L_8 - t_{7,8} = 17 - 2 = 15$

$L_6 = L_8 - t_{6,8} = 17 - 1 = 16$

$L_5 = Min\{L_j - t_{5,j}\}$

$= Min\{L_6 - t_{5,6} ; L_7 - t_{5,7}\}$

$= Min\{16-4 ; 15-8\}$

$= Min\{12 ; 7\}$

$= 7.$

$L_4 = L_9 - t_{4,9} = 15 - 5 = 10$

$L_3 = Min\{L_j - t_{3,j}\}$

$= Min\{L_4 - t_{3,4} ; L_5 - t_{3,5}\}$

$= Min\{10-1 ; 7-6\}$

$= Min\{9, 1\}$

$= 1$

$L_2 = L_4 - t_{2,4} = 10 - 1 = 9$

$L_1 = min\{L_2 - t_{1,2} ; L_3 - t_{1,3}$

$= Min\{9-4 ; 1-1\}$

$= min\{5 ; 0\}$

$= 0$

The Critical path of the project is:

1 - 3 - 5 - 7 - 8 - 10

Critical activities are A1, A4, A7, A9, A10

The total Project time is <u>22</u>

(P.T.O)

Activity (i,j) (1)	Duration t_{ij} (2)	Earliest start E_i & E_j: E_i (3)	E_j (4)	Latest Finish: L_i (5)	L_j (6)	Earliest Finish $= E_i + t_{ij}$ (7)	Latest start $= L_j - t_{ij}$ (8)	Total float $= L_j - t_{ij} - E_i$ (9) = (8) - (3)	Free float $= E_j - E_i - t_{ij}$ (10) = (4) - (3) - (2)	Independent float $= E_j - L_i - t_{ij}$ (11) = (4) - (5) - (2)
1-2	4	0	4	0	9	4	5	5	4-0-4=0	4-0-4=0
1-3	1	0	1	0	1	1	0	0	1-0-1=0	1-0-1=0
2-4	1	4	5	9	10	5	9	5	5-4-1=0	5-9-1=-5
3-4	1	1	5	1	10	2	9	8	5-1-1=3	5-1-1=3
3-5	6	1	7	1	7	7	1	0	7-1-6=0	7-1-6=0
4-9	5	5	10	10	15	10	10	5	10-5-5=0	10-10-5=-5
5-6	4	7	11	7	16	11	12	5	11-7-4=0	11-7-4=0
5-7	8	7	15	7	15	15	7	0	15-7-8=0	15-7-8 0
6-8	1	11	17	16	17	12	16	5	17-11-1=5	17-16-1=0
7-8	2	15	17	15	17	17	15	0	17-15-2=0	17-15-2=0
8-10	5	17	22	17	22	22	17	0	22-17-5=0	22-17-5=0
9-10	7	10	22	15	22	17	15	5	22-10-7=5	22-15-7=0

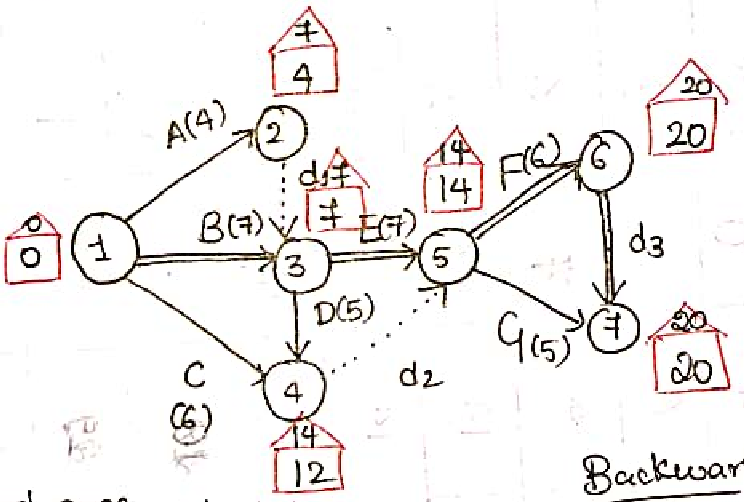
2024 8. A small project consists of 7 activities for which the relevent data are given below.

Activity	Preceding activities	Activity duration
A	-	4
B	-	7
C	-	6
D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5

i) Draw the network & find the project completion time.

ii) Calculate total float for each of the activities.

Sol:-



Forward pass calculation

$E_1 = 0$

$E_2 = 0+4 = 4$

$E_3 = Max\{4+0, 0+7\}$
$= 7$

$E_4 = Max\{0+6, 7+5\}$
$= 12$

$E_5 = 7+7 = 14$

$E_6 = 14+6 = 20$

$E_7 = max\{5+14, 20+0\} = 20$

Backward pass calculation

$L_7 = E_7 = 20$

$L_6 = 20-0 = 20$

$L_5 = min\{15, 14\}$
$= 14$

$L_4 = 14-0$

$L_3 = Min(14-5, 14-7)$
$= 7$

$L_2 = 7-0 = 7$

$L_1 = Min(7-4, 7-7, 14-6)$
$= 0$

The Critical path of a project is,

1-3-5-6-7.

Critical activities are :- B, E, F, & G

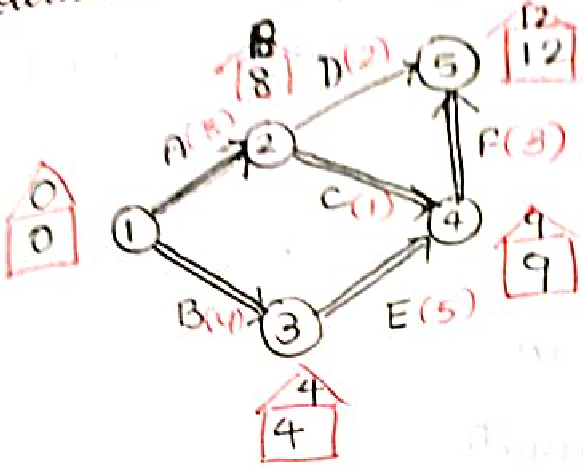
The total project time is 20

Activity (i-j) (1)	Duration t_{ij} (2)	Earliest Start E_i (3)	Earliest Start E_j (4)	Latest Finish L_i (5)	Latest Finish L_j (6)	Earliest Finish $= E_i + t_{ij}$ (7)	Latest Start $= L_j - t_{ij}$ (8)	Total float $= L_j - t_{ij} - E_i$ (9)	Free float $= E_j - E_i - t_{ij}$ (10)	Independent float $= E_j - L_i - t_{ij}$
1-2	4	0	4	0	7	4	3	3	4-0-4=0	4-0-4=0
1-3	7	0	7	0	7	7	0	0	7-0-7=0	7-0-7=0
1-4	6	0	12	0	14	6	8	8	12-0-6=6	12-0-6=6
2-3	0	4	7	7	7	4	7	3	7-4-0=3	7-7-0=0
3-4	5	7	12	7	14	12	9	2	12-7-5=0	12-7-5=0
3-5	7	7	14	7	14	14	7	0	14-7-7=0	14-7-7=0
4-5	0	12	14	14	14	12	14	2	14-12-0=2	14-14-0=0
5-6	6	14	20	14	20	20	14	0	20-14-6=0	20-14-6=0
5-7	5	14	20	14	20	19	15	1	20-14-5=1	20-14-5=1
6-7	0	20	20	20	20	20	20	0	20-20-0=0	20-20-0=0

8. Activity: 1-2, 1-3, 2-4, 2-5, 3-4, 4-5

Duration: 8, 4, 1, 2, 5, 3

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration	8	4	1	2	5	3



The total project time is 12

Activity (i-j)	Duration t_{ij}	Earliest Start E_i	E_j	Latest finish L_i	L_j	Earliest Finish $= E_i + t_{ij}$	Latest Start $= L_j - t_{ij}$	TF $= L_j - t_{ij} - E_i$	FF $= E_j - E_i - t_{ij}$	IF $= E_j - L_i - t_{ij}$
1-2	8	0	8	0	8	8	0	0	0	0
1-3	4	0	4	0	4	4	0	0	0	0
2-4	1	8	9	8	9	9	8	0	0	0
2-5	2	8	12	8	12	10	10	2	2	2
3-4	5	4	9	4	9	9	4	0	0	0
4-5	3	9	12	9	12	12	9	0	0	0

PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

PERT is a probabilistic method, where the activity times are represented by a probability distribution. This distribution of activity times is based on three different time estimates made for each activity, which are as follows:

(i) Optimistic time estimate

(ii) Most likely time estimate

(iii) Pessimistic time estimate.

Optimistic time estimate :- It is the smallest time taken to complete the activity, if everything goes well. There is very little chance that an activity can be completed in a time less than the Optimistic time. It is denoted by t_o or a.

Most likely time estimate:- It refers to the estimate of the normal time the activity would take. It is the mode of the probability distribution. It is denoted by t_m or m.

Pessimistic time estimate:- It is the longest time that an activity would take, if everything goes wrong. It is denoted by t_p or b.

$$\text{Expected time}, \; t_e = \frac{t_o + 4t_m + t_p}{6}$$

Variance of the activity is given by,

$$\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$$

The main objective of the analysis through PERT is to find the completion date for a particular event within the specified date T_s, given by $P(Z \leq D)$ where,

$$D = \frac{\text{Due date} - \text{Expected date of completion}}{\sqrt{\text{Project variance}}}$$

Here, Z - Standard normal variable.

$$Z = \frac{T_S - T_e}{\sigma}$$

ex:- The following table shows the jobs of a network along with their time estimates.

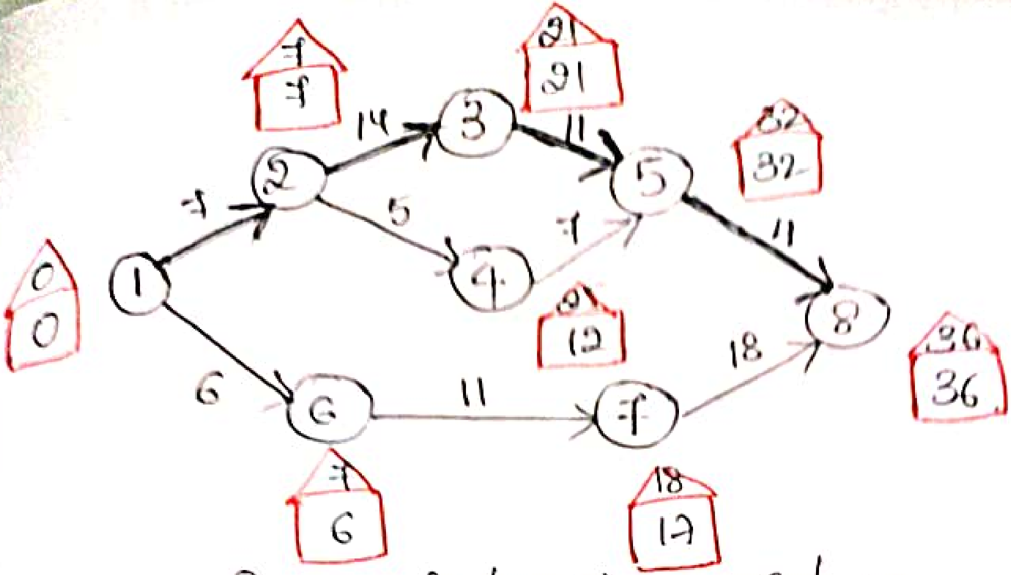
Job	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
a (days)	1	2	2	2	7	5	5	3	8
m (days)	7	5	14	5	10	5	8	3	17
b (days)	13	14	26	8	19	17	29	9	32

Draw the project network and find the probability of the project completing in 40 days.

(P.T.O)

First we calculate the expected time & standard deviation for each activity.

Activity	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	$\frac{1+(4\times7)+13}{6} = 7$	$\left(\frac{13-1}{6}\right)^2 = 4$
1-6	$\frac{2+(4\times5)+14}{6} = 6$	$\left(\frac{14-2}{6}\right)^2 = 4$
2-3	$\frac{2+(4\times14)+26}{6} = 14$	$\left(\frac{26-2}{6}\right)^2 = 16$
2-4	$\frac{2+(5\times4)+8}{6} = 5$	$\left(\frac{8-2}{6}\right)^2 = 1$
3-5	$\frac{7+(4\times10)+19}{6} = 11$	$\left(\frac{19-7}{6}\right)^2 = 4$
4-5	$\frac{5+(5\times4)+17}{6} = 7$	$\left(\frac{17-5}{6}\right)^2 = 4$
6-7	$\frac{5+(8\times4)+29}{6} = 11$	$\left(\frac{29-5}{6}\right)^2 = 16$
5-8	$\frac{3+(3\times4)+9}{6} = 4$	$\left(\frac{9-3}{6}\right)^2 = 1$
7-8	$\frac{8+(4\times17)+32}{6} = 18$	$\left(\frac{32-8}{6}\right)^2 = 16.$



Expected project duration = 36 days.

Critical path 1-2-3-5-8.

Project length variance, $\sigma^2 = 4+16+4+1 = 25$

Std. Deviation, $\sigma = 5$.

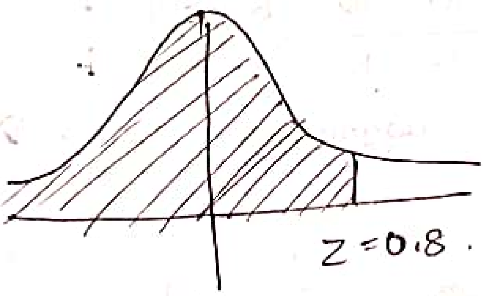
The probability that the project will be completed in 40 days is given $P(Z \leq D)$

$$D = \frac{T_s - T_e}{\sigma} = \frac{40-36}{5} = \frac{4}{5} = 0.8.$$

Area under the normal curve for the region

$Z \leq 0.8$

$$P(Z \leq 0.8) = 0.5 + \phi(0.8)$$
$$= 0.5 + 0.2881$$
$$= 0.7881$$
$$= 78.81\%.$$

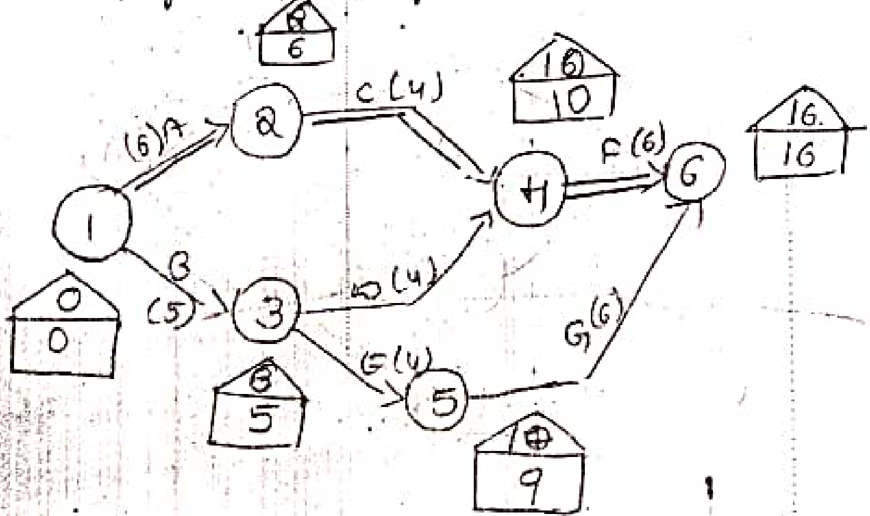


$\phi(8) = 0.2881$

Problem 3

Activity	estimated duration t_o opti(a)	t_m most (m)	t_p Pess(b)	Predecessor	t_e	σ^2
A	3	6	9	—	6	1
B	2	5	8	—	5	1
C	2	4	6	A	4	0.4
D	2	3	10	B	4	1.77
E	1	3	11	B	4	2.76
F	4	6	8	C, D	6	0.444
G	1	5	15	E	6	5.444

Find the path and standard deviation Also find the probability of Completing the project by 18 week



Expected Complete time = 18 week

Critical path → ~~A-2, 2-4, 4-6~~ A, C, F

Project length varience → 1 + 0.4 + 0.444

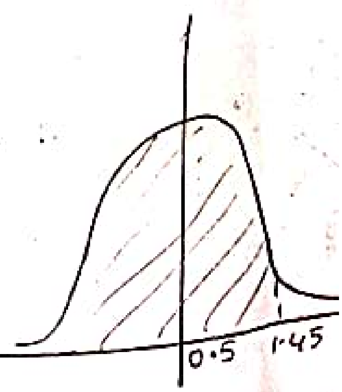
σ^2 → 1.888

σ = 1.374

T_e → 16 weeks

T_s → 18 weeks

$$D = \frac{18-16}{1.374} = \frac{2}{1.374} = 1.4556$$



$P(z \leq D)$

$0.5 + \phi(1.4556)$

$= 0.5 + 0.4265$

$= 0.926$

$= 92.6\%$

PERT	CPM
Full form :- Project Evaluation & Review Technique.	Full form :- Critical Path method.
It is Event oriented technique	It is activity oriented technique
PERT manages unpredictable activities	CPM manages the predictable activities.
It is focused on time control	It is focused on cost optimization
It is a probability model	It is a deterministic model
Three time estimates	One time estimate.